THE EFFECT OF TRANSPORT PROCESSES ON THE STABILITY OF A PLANE FLAME FRONT

(O VLIANII PROTSESSOV PERENOSA NA USTOICHIVOST PLOSKOGO FRONTA PLAMENI)

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The absolute instability of a plane front of a laminary flame was proved by Landau [1 and 2] on the assumption that the Reynolds number, defined with respect to the perturbation wave length, was infinitely large. This assumption made it possible for him, when analysing stability, to utilize the equations of a perfect fluid and to consider the flame front as an infinitely thin hydrodynamic discontinuity propagating in relation to the gas with a known constant velocity normal to its front.

This work deals with the determination of the next approximation to Landau's problem with respect to a small parameter reciprocal to the Reynolds number. In this approximation and with the assumption that the Reynolds number is large, but not infinitely large, it is necessary to take into account the transport phenomena (viscosity, diffusion and heat conductivity), because these affect the flame stability by altering the field of gas flow outside the flame zone, and determine the flame structure itself. The changes in the rate of chemical reaction within the flame must also be taken into account, as these affect the flame propagation velocity relative to the combustive mixture.

The results obtained for a simple thermal mechanism of flame propagation permit the determination of the critical wavelength of perturbations, and of the critical Reynolds number at which a laminar flame is stable. The latter depends on the degree of thermal expansion of the burning gas, on the Prandtl and Lewis numbers, and on the dimensionless energy of activation of the chemical reaction.

We would point out that the analysis of the influence of the stabilising factors on a laminar flame, as given in [3 to 9], suffers from substantial defects. Authors of [3, 4 and 6] have shown a lack of understanding of the asymptotic character of the Landau's theory of stability, and that led to inconsistencies in the computation of corrections of flame perturbations at finite Reynolds numbers. This remark refers in particular to the derivation of boundary conditions at the flame surface. In none of the quoted works was the finite thickness of the flame front taken into account. This was due to the lack hitherto of a mathematical device for dealing with iterative computation of corrections to solutions possessing discontinuities at zero-order approximations. Only recently have Germain and

Guiraud suggested in their work on slightly curved shock waves [10 to 12] a method of iterative computation of corrections of a given order in boundary conditions on the hydrodynamic discontinuities. This method is used here for the flame-front analysis.

Furthermore, attempts made in the published works dealt only with some of the factors affecting stability, while ignoring others of the same order of magnitude (for example, in [3, 6 and 7] diffusion and thermal conductivity were taken into account, but not viscosity, and vice-versa in [5]). We would stress that in the method of successive approach to this problem it is necessary to consider jointly the following factors which are of the same order of magnitude, namely $\varepsilon = 1 / N_R$, (here $(N_R = u_n / k\nu$ is the Reynolds number, u_n is the flame propagation velocity, k is the wave perturbation number, and ν the viscosity). These factors are: (1) effect of the curvature of the flame front on the processes of diffusion, heat conductivity and viscosity within the flame front itself; (2) the effect of temperature perturbation on the rate of combustion (rate of chemical reaction in the flame); (3) the effect of the finite width (thickness) of the flame front; (4) the effect of temperature perturbations on density changes upstream of the flame front; (5) the effect of viscosity on the motion of gas outside the flame front. All these effects are taken into consideration in the present work.

The system of notations used in this work is as follows: Superscripts ^{0} and ^{1} refer respectively to the zero-, and first-order approximations with respect to ε. The subscripts (–) and (+) denote the solutions applicable in the zone of the combustible mixture, and the zone of products of combustion respectively. During the linearisation, the parameters describing the stationary unperturbed solution are denoted by capitals, and those pertaining to corresponding perturbations, by italics with primes. Thus, for the unperturbed state U denotes the velocity, R the density, P the pressure, Θ the dimensionless temperature, and S the concentration while u', ρ', p', θ', and s' represent the corresponding perturbations. Italics without primes denote full values of relevant parameters $(\rho = R + \rho')$. Other notations will be defined in the following text.

1. We shall derive corrections of the order of $1/N_R$ to the Landau's solution by the method suggested by Germain and Guiraud in [10 to 12] for the assessment of transport processes in slightly curved shock waves.

According to this method it is necessary to distinguish two areas of gas flow, one outside the flame zone proper, the other within it (streamlines of a gas flowing through a curved front of a laminar flame are shown on fig. 1). In the external area of flow the gradients of parameters are small (if these parameters are dimensionally related to the perturbation wavelength and the normal velocity of flame propagation, then these gradients are ~ 1), and the usual iterative methods can be applied. In other words, such parameters can be presented in the form

$$f = f^0 + \varepsilon f^1 \tag{1.1}$$

where f^0 is the solution in the zero-order approximation (Landau's solution), and f^1 is the correction which is found from the relevant formulas, by substituting expressions of the form of (1.1) into them.

The iteration method is not applicable to the internal region of the flame the width of which is of the order of $\varepsilon = 1 / N_R$, since it is also affected by transport phenomena. The parameter gradients are large, viz. $\sim 1 / \varepsilon$.

However, such transport processes can be assessed for this region, if its narrowness compared to the dimensions of the region of perturbed flow outside it, is taken into account. For this purpose it will be necessary to formulate boundary conditions for the solutions in the external zones, which would take into consideration the structure and the width of the flame front, i.e. which would present the influence of the inner zone effects in an integrated form. At the same time, the boundary conditions at the flame surface will, in general, be expressed by

$$[f^0] + \varepsilon\,[f^1] = \varepsilon\Delta\,(f^0) \tag{1.2}$$

where $[f^0]$ is the parametric discontinuity at the flame front corresponding to the zero-order approximation to the solution of this problem, and $[f^1]$ and $\Delta[f^0]$ are the corrections related to the transport processes and to the finite width of the flame-front.

Having established the relationship between the solutions for the cold gas zone and the the zone of combustion products of the form of (1.1) by means of the boundary conditions (1.2), we can equate the coefficients of the zero-, and first-power of ε, to obtain two systems of homogeneous linear algebraic equations. The first (zero-order approximation) is the Landau solution which will be used for deriving the characteristic frequency of Landau's theory and for correlating the amplitudes of pressure and velocity perturbation on both sides of the flame zone, with the amplitude of the perturbation of the flame front itself. The second system (first-order approximation) makes possible the determination of the amplitudes of the first-order approximation in terms of the amplitude of perturbation of the flame surface, and the determination of the change of the characteristic frequency of the problem. Obviously, in the first approximation solution, the variation of the characteristic frequency should be of the type

$$\omega = \omega^0\,[1 - \varepsilon\tau\,(\sigma, L, \alpha, z)] \qquad (z = E/2R_0T_b) \tag{1.3}$$

Here ω is the dimensionless characteristic frequency, ω^0 is the zero-order approximation of the characteristic frequency, τ is the first-order correction, α is the ratio of the densities of the hot and cold gases, σ is the Prandtl number, L is the ratio of thermal conductivity of the gas to the coefficient of diffusion of the burning gas (the Lewis number), z is the dimensionless energy of activation which determines the dependence of the rate of chemical reaction in the flame in temperature, E is the activation energy, and T_b is the temperature of combustion products. It is assumed that the mechanism of flame propagation is thermal, such and that the rate of reaction depends on the combustible component only.

The problem is thus reduced to determination of the function $\tau\,(\sigma, L, \alpha, z)$.

Strictly speaking, solution of the frequency problem (1.3) will only indicate the trend of the change of the natural frequency of perturbation at finite Reynolds numbers. In order to change the sign of the frequency, it is necessary to extrapolate the expression (1.3) to such wavelengths of perturbation for which $\varepsilon\tau\,(\sigma, L, \alpha, z) \sim 1$ (in this case τ represents the critical Reynolds number).

We note that the same equations of conservation of mass, reactant, energy and momentum can be used for both the iteration in the area outside the flame zone, and for the derivation of boundary conditions at the flame front. Landau had used in his work the equations of conservation of mass and momentum only. This is because in the case of the zero-order approximation to the solution of flame stability, the equations of reactant and energy conservation are reduced to trivial equations of temperature and reactant transfer by the stream of gas; the solutions of such equations derived for perturbations of the reactant concentration are identically equal to zero.

Such equations may, therefore, be omitted from further considerations. At the flame boundary there remains only an insignificant term of the equation of reactant conservation, viz. the condition for the normal flame velocity to be constant (which is in fact the condition of a constant rate of consumption of the reactant by the chemical reaction). In the next following approximation to the solution of the flame stability problem it will be necessary to take into consideration the transfer of energy, matter and momentum by means of thermal conductivity, diffusion and viscosity. Here, the equations of energy and reactant conservation will not lead to trivial solutions, and must be taken into account.

2. The fundamental equations of this problem are the equations of conservation of mass, momentum, energy, and reactant.

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\varepsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\varepsilon}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\varepsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{\varepsilon}{3}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

$$\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}=-\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\tag{2.2}$$

$$\rho\frac{\partial\theta}{\partial t}+\rho u\frac{\partial\theta}{\partial x}+\rho v\frac{\partial\theta}{\partial y}=\frac{\varepsilon}{\sigma}\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+\frac{w}{\varepsilon}\qquad\left(\theta=\frac{T-T_0}{T_b-T_0}\right)$$

$$\rho\frac{\partial s}{\partial t}+\rho u\frac{\partial s}{\partial x}+\rho v\frac{\partial s}{\partial y}=\frac{\varepsilon}{\sigma L}\left(\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}\right)-\frac{w}{\varepsilon}$$

The following notations are used in these equations: x, y denote dimensionless plane coordinates (see fig. 1) related to the perturbation wavelength, or more precisely to $k=2\pi/\lambda$, (k is the wave number), t is the dimensionless time defined by the ratio of actual time to the characteristic time $1/u_n k$, u and v are the dimensionless components of velocity in units of normal flame propagation velocity u_n, p is the dimensionless pressure (ratio of pressure to the dynamic pressure of the fresh mixture), ρ is the dimensionless density (relative to cold gas density), s is the relative concentration of the reactant, θ is the dimensionless temperature, T_0 is the initial temperature of the mixture, T_b is the temperature of products of combustion under the stationary conditions, and w is the dimensionless rate of the chemical reaction. It is also assumed that the thermal conductivity, dynamic viscosity, specific heat, and the product of the coefficient of diffusion and density remain constant, and independent of either temperature, or concentration.

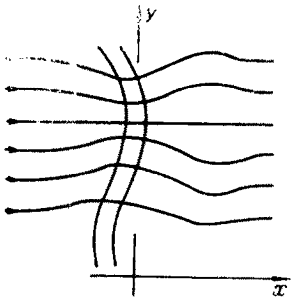

FIG. 1

From the last of Equations (2.1) we can see, that the expression for the chemical reaction rate exibits the parameter ε in a separate term. The reason for its appearance in the denominator becomes clear from the following considerations. From the general considerations of the theory of dimensional analysis it follows that the normal velocity of flame propagation u_n is related to the thermal conductivity of the gas $\varkappa$ and to the chemical reaction rate $W(T, s)$ by the expression $u_n \sim (\varkappa W)^{1/2}$, or is equivalent

$W \sim u_n^2 / \varkappa$. By dimensioning this relationship with respect to the normal flame velocity and the perturbation wave number, we arrive at the form utilised in (2.1). This form clearly reflects the obvious fact, that with ε tending to zero, the heat generation rate must become a δ-function related to an infinitely thin flame front.

Equations (2.1) must be supplemented by equations of state of the gas which, in the case of an incompressible gas (flame velocity small compared with the velocity of sound), is reduced to the relation between the density of the gas and its temperature

$$\rho = 1 \Big/ \left(1 + \frac{1-\alpha}{\alpha}\theta\right) \tag{2.2}$$

The analysis of stability will be carried out by the method of small perturbations, and the solution of Equations (2.1) will be sought in the form

$$f = F + f' = F(x) + [f^0(x) + \varepsilon f^1(x)] \exp(iy + \omega t) \tag{2.3}$$

We shall substitute solutions of the form of (2.3) into our equations and retain the terms of the order of ε. Eliminating the common factor $\exp(iy + \omega t)$ we obtain

$$\begin{gathered}
R\omega u^0 + \varepsilon R\omega u^1 + \frac{\partial u^0}{\partial x} + \varepsilon\frac{\partial u^1}{\partial x} = -\frac{\partial p^0}{\partial x} - \varepsilon\frac{\partial p^1}{\partial x} + \varepsilon\left(\frac{\partial^2 u^0}{\partial x^2} - u^0\right) + \\
+ \frac{\varepsilon}{3}\frac{\partial}{\partial x}\left(\frac{\partial u^0}{\partial x} + iv^0\right) \\
R\omega v^0 + \varepsilon R\omega v^1 + \frac{\partial v^0}{\partial x} + \varepsilon\frac{\partial v^1}{\partial x} = -ip^0 - i\varepsilon p^1 + \varepsilon\left(\frac{\partial^2 v^0}{\partial x^2} - v^0\right) + i\frac{\varepsilon}{3}\left(\frac{\partial u^0}{\partial x} + iv^0\right) \\
\omega R(\theta^0 + \varepsilon\theta^1) + \frac{\partial}{\partial x}(\theta^0 + \varepsilon\theta^1) = \frac{\alpha}{1-\alpha}\left[\frac{\partial}{\partial x}(u^0 + \varepsilon u^1) + i(v^0 + \varepsilon v^1)\right] \\
R\omega\theta^0 + \varepsilon R\omega\theta^1 + \frac{\partial \theta^0}{\partial x} + \varepsilon\frac{\partial \theta^1}{\partial x} = \frac{\varepsilon}{\sigma}\left(\frac{\partial^2\theta^0}{\partial x^2} - \theta^0\right) \\
R\omega s^0 + \varepsilon R\omega s^1 + \frac{\partial s^0}{\partial x} + \varepsilon\frac{\partial s^1}{\partial x} = \frac{\varepsilon}{\sigma L}\left(\frac{\partial^2 s^0}{\partial x^2} - s^0\right)
\end{gathered} \tag{2.4}$$

We have eliminated the factor $\rho^0 + \varepsilon\rho^1$ from the continuity equation by means of the expression (2.2). Also, terms w / ε have been omitted from the last equations, since outside the flame the rate of reaction is zero.

In order to solve the above equations with zero-order approximation with respect to ε (the Landau solution), we assume $\varepsilon = 0$ and utilise the following boundary conditions.

$$\begin{gathered}
u_-^{0,1},\ v_-^{0,1},\ p_-^{0,1},\ \theta_-^{0,1},\ s_-^{0,1} \to 0 \quad \text{when } x \to -\infty \quad \text{(cold gas)} \\
u_+^{0,1},\ v_+^{0,1},\ p_+^{0,1},\ s_+^{0,1} \to 0, \quad |\theta_+^{0,1}| < \infty \text{ when } x \to +\infty \text{ (combustion products)}
\end{gathered} \tag{2.5}$$

For the cold gas zone ($x < 0$) we have

$$\begin{gathered}
u_-^0 = A^0 e^x, \qquad v_-^0 = iA^0 e^x, \qquad p_-^0 = -A^0(\omega + 1)e^x \\
\theta_-^0 = s_-^0 = \rho_-^0 = 0 \qquad (A^0 - \text{ is a constant of integration})
\end{gathered} \tag{2.6}$$

To find the terms of the order of ε we must equate the coefficients of ε in Equations (2.4) and substitute the expressions derived for the zero-order approximations into them. This gives for the cold gas zone the following equations for u_-^1, v_-^1, p_-^1, θ_-^1, and s_-^1

$$\omega u_-^1 + \frac{\partial u_-^1}{\partial x} = -\frac{\partial p_-^1}{\partial x}, \qquad \omega v_-^1 + \frac{\partial v_-^1}{\partial x} = -ip_-^1$$
$$\frac{1-\alpha}{\alpha}\left(\omega\theta_-^1 + \frac{\partial\theta_-^1}{\partial x}\right) = \frac{\partial u_-^1}{\partial x} + iv_-^1, \quad \omega\theta_-^1 + \frac{\partial\theta_-^1}{\partial x} = 0, \quad \omega s_-^1 + \frac{\partial s_-^1}{\partial x} = 0 \tag{2.7}$$

The solution of these equations with boundary conditions (2.5) is identical to that of the zero-approximation (2.6), except for the new constant of integration A^1.

It follows that even in the first order approximation with respect to ε the perturbations of temperature, concentration and density in the cold gas zone are absent. The transfer of the reactant and of heat energy is effected by convection only, while the processes of diffusion and thermal conductivity become substantial in the second-order approximation.

The solution of Equations (2.4) of the gas flow downstream of the flame front ($x > 0$), (in the region of hot combustion products) yields the following results

$$u_+^0 = B^0e^{-x} + C^0e^{-\alpha\omega x}, \qquad v_+^0 = -iB^0e^{-x} - i\alpha\omega C^0e^{-\alpha\omega x}$$
$$p_+^0 = B^0(\alpha\omega - 1)e^{-x}, \qquad \theta_+^0 = H^0e^{-\alpha\omega x}, \qquad \rho_+^0 = -\alpha(1-\alpha)\theta_+^0 \tag{2.8}$$
$$s_+^0 = 0 \qquad (B^0,\ C^0,\ H^0 \text{ — are the constants of integration})$$

We draw the attention to the following circumstances. The flow of gas downstream of the flame front is turbulent (as opposed to the flow upstream of it), vortices form on the flame surface and are then carried away by the gas stream (in the expressions for the velocity components the terms with the constant C^0 correspond to the turbulent motion of gas). Furthermore, temperature and related density perturbations downstream of the flame front may be caused by temperature perturbations originating at the flame front and transferred by convection.

In the first-order approximation equations of the hot gas zone have the form

$$\alpha\omega u_+^1 + \frac{\partial u_+^1}{\partial x} = -\frac{\partial p_+^1}{\partial x} + [(\alpha\omega)^2 - 1]\,C^0e^{-\alpha\omega x}$$
$$\alpha\omega v_+^1 + \frac{\partial v_+^1}{\partial x} = -ip_+^1 + i\alpha\omega\,[1 - (\alpha\omega)^2]\,C^0e^{-\alpha\omega x}$$
$$\frac{1-\alpha}{\alpha}\left(\alpha\omega\theta_+^1 + \frac{\partial\theta_+^1}{\partial x}\right) = \frac{\partial u_+^1}{\partial x} + iv_+^1, \quad \alpha\omega\theta_+^1 + \frac{\partial\theta_+^1}{\partial x} = \frac{1}{\sigma}\,[(\alpha\omega)^2 - 1]\,H^0e^{-\alpha\omega x} \tag{2.9}$$
$$\rho_+^1 = -\alpha(1-\alpha)\theta_+^1, \qquad s_+^1 = 0$$

the solutions of which are

$$u_+^1 = B^1e^{-x} + C^1e^{-\alpha\omega x} + [(\alpha\omega)^2 - 1]\,C^0xe^{-\alpha\omega x}$$
$$v_+^1 = -iB^1e^{-x} - i\alpha\omega C^1e^{-\alpha\omega x} - i\,[1 - (\alpha\omega)^2]\,(1 - \alpha\omega x)\,C^0e^{-\alpha\omega x}$$
$$p_+^1 = B^1(\alpha\omega - 1)e^{-x}, \quad \rho_+^1 = -\alpha(1-\alpha)\theta_+^1 \tag{2.10}$$
$$\theta_+^1 = H^1e^{-\alpha\omega x} + \frac{1}{\sigma}\,[(\alpha\omega)^2 - 1]\,H^0xe^{-\alpha\omega x}$$

Here B^1, C^1 and H^1 are new constants of integration. These equations contain terms representing the transfer of heat energy by conduction, as well as the effect of viscosity downstream of the flame front.

Finally, we shall define the form of the curved flame front by

$$\zeta = D\exp(iy + \omega t) \tag{2.11}$$

The computation of the constants of integration of these solutions is carried out with the aid of boundary conditions at the flame surface which correlate the solutions of the hot and cold gas zones. These boundary conditions must also be calculated in the first-order approximation with respect to ε.

3. Conditions of conservation of the longitudinal and transverse components of impulse, mass, heat energy, and reactant concentration must be maintained at the flame surface (the two last conditions must take into account the heat generated by the flame and the consumption of the reactant by it). Insofar as these conditions which ecpress the laws of conservation appear in a differential form in Equations (2.1), the latter can also be used for the derivation of boundary conditions.

We shall illustrate the method of derivation of boundary conditions on the example of conservation of the longitudinal component of impulse. Let us rewrite the first of Equations (2.1) in the divergent form

$$\frac{\partial \rho u}{\partial t} = \frac{\partial}{\partial x}\left[-p - \rho u^2 + \varepsilon\left(\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right)\right] + \frac{\partial}{\partial y}\left[-\rho uv + \varepsilon\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] \tag{3.1}$$

We shall integrate this equation with respect to x from $\zeta - \delta$ to $\zeta + \delta$ (in the proximity of the flame surface). In order to include the whole of the flame structure in the limits of integration we select $\delta \gg \varepsilon$. Applying the rules of differentiating under the integral sign with respect to a parameter, we obtain

$$\begin{aligned}\frac{\partial}{\partial t}\int_{\zeta-\delta}^{\zeta+\delta} \rho u\, dx - [\rho u]_{\zeta-\delta}^{\zeta+\delta}\frac{\partial \zeta}{\partial t} &= \left[-p - \rho u^2 + \varepsilon\left(\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right)\right]_{\zeta-\delta}^{\zeta+\delta} + \\ + \frac{\partial}{\partial y}\int_{\zeta-\delta}^{\zeta+\delta}\left[-\rho uv + \varepsilon\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] dx &- \left[-\rho uv + \varepsilon\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]_{\zeta-\delta}^{\zeta+\delta}\frac{\partial \zeta}{\partial y}\end{aligned} \tag{3.2}$$

Notations $[\ ]_{\zeta-\delta}^{\zeta+\delta}$ are used here for denoting differences of the bracketed terms at the two sides of the flame.

Expression (3.2) contains terms of three kinds. Firstly, we have the usual differences of the impulse stream along a moving curved surface

$$[\rho u]_{\zeta-\delta}^{\zeta+\delta}\frac{\partial \zeta}{\partial t}, \qquad [-p - \rho u^2]_{\zeta-\delta}^{\zeta+\delta}, \qquad [-\rho uv]_{\zeta-\delta}^{\zeta+\delta}\frac{\partial \zeta}{\partial y} \tag{3.3}$$

Secondly, we have corrections of the order ε, caused by the presence of a viscous impulse stream on the boundaries $\zeta - \delta$ and $\zeta + \delta$

$$\varepsilon\left[\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right]_{\zeta-\delta}^{\zeta+\delta}, \qquad -\varepsilon\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]_{\zeta-\varepsilon}^{\zeta+\delta}\frac{\partial \zeta}{\partial y} \tag{3.4}$$

These corrections are obtained by computing the relevant derivatives of solutions for the ideal gas (formulas (2.6) and (2.8).

Thirdly, we have the following terms:

$$\frac{\partial}{\partial t}\int_{\zeta-\delta}^{\zeta+\delta} \rho u\, dx, \qquad \frac{\partial}{\partial y}\int_{\zeta-\delta}^{\zeta+\delta}\left[-\rho uv + \varepsilon\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] dx \tag{3.5}$$

Germain and Guiraud were the first to note [10 to 12] that these integrals also yield corrections of the order of ε, but to be able to compute them, we must know the flame structure at the zero-order approximation with respect to ε.

We shall explain this in the manner given in [10 to 12]. Any parameter f (density, velocity, temperature, etc.) discontinuous at $\varepsilon = 0$, can be represented for the finite values of ε in the form

$$f = \frac{1}{2}[f_+(x-\zeta, y, t, \varepsilon) + f_-(x-\zeta, y, t, \varepsilon)] + \frac{1}{2}[f_+(x-\zeta, y, t, \varepsilon) - \\ - f_-(x-\zeta, y, t, \varepsilon)]\,\Phi\left(\frac{x-\zeta}{\varepsilon}, y, t, \varepsilon\right) \tag{3.6}$$

where $\Phi(\pm\infty, y, t, \varepsilon) = \pm 1$. At some distance from the discontinuity where $|x-\zeta| \gg \varepsilon$, function f must be congruent with the solutions in the discontinuity interval, i.e. with f_+ for $x - \zeta > 0$ and with f_- for $x - \zeta < 0$ (it is assumed here that f tends to $f_\pm$ exponetially). This function also defines the structure of the discontinuity at $|x-\zeta| \lesssim \varepsilon$, since by definition, ε defines the width of the discontinuity.

From this we see that the integration of f with respect to x outside the discontinuity zone will yield real values only for f_+ and f_- while within the zone, the result of integration may be expressed by εf^*, where f^* is a certain effective value of parameter f within the discontinuity, and which depends on its structure. For the computation of f^* Germain and Guiraud have suggested a method similar to the concept of the thickness of displacement of the boundary layer theory. In accordance with this method we shall express the function f in the form of $f_e + f - f_e$, where for $x - \zeta > 0$ we have $f_e = f_+$, and for $x - \zeta < 0$, $f_e = f_-$. Then

$$\int_{\zeta-\delta}^{\zeta+\delta} f\,dx = \int_{\zeta-\delta}^{\zeta+\delta} f_e\,dx + \int_{\zeta-\delta}^{\zeta+\delta} (f - f_e)\,dx \tag{3.7}$$

The first of these integrals is always present in the formulation of boundary conditions at an infinitesimally thin discontinuity. The integrand of the second integral differs from zero due to the discontinuity possessing a definite structure. In order to compute this integral we make the substitution $\xi = (x-\zeta)/\varepsilon$ to obtain

$$\int_{\zeta-\delta}^{\zeta+\delta} (f - f_e)\,dx = \varepsilon \int_{-\delta/\varepsilon}^{\delta/\varepsilon} (f - f_e)\,d\xi \tag{3.8}$$

Since outside the discontinuity $f - f_e$ tends exponentially to zero, and $\delta \gg \varepsilon$, the limits of integration can be extended to from $-\infty$ to $+\infty$. Consequently

$$f^* = \int_{-\infty}^{\infty} (f - f_e)\,d\xi \tag{3.9}$$

The boundary condition (3.2) correlates the parameters at $x = \zeta + \delta$ and $x = \zeta - \delta$. In order to obtain the conditions at the surface of the discontinuity, we expand all the parameters into series in terms of δ, and take the first terms of expansions.

Thus, the conditions of conservation of the flow of the longitudinal component of the impulse is expressed by

$$\varepsilon \frac{\partial}{\partial t}(\rho u)^* - [\rho u]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial t} = \left[-p - \rho u^2 + \varepsilon\left(\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right)\right]_{\zeta-0}^{\zeta+0} - \\ - \varepsilon\frac{\partial}{\partial y}(\rho u v)^* - \left[-\rho u v + \varepsilon\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial y} \tag{3.10}$$

We obtain in a similar manner conditions of continuity of the flow of the lateral component of impulse, and the condition of conservation of the mass flow.

$$\varepsilon \frac{\partial}{\partial t}(\rho v)^* - [\rho v]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial t} = \left[-\rho u v + \varepsilon\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right]_{\zeta-0}^{\zeta+0} -$$

$$- \varepsilon \frac{\partial}{\partial y}(p + \rho v^2)^* - \left[-p - \rho v^2 + \varepsilon\left(\frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial u}{\partial x}\right)\right]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial y} \quad (3.11)$$

$$\varepsilon \frac{\partial}{\partial t}\rho^* - [\rho]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial t} + [\rho u]_{\zeta-0}^{\zeta+0} + \varepsilon\frac{\partial}{\partial y}(\rho v)^* - [\rho v]_{\zeta-0}^{\zeta+0}\frac{\partial \zeta}{\partial y} = 0$$

Equations of heat and reactant transfer can be dealt with in the same manner. Boundary conditions for these will contain the integral of the function of emission of heat

$$M = \int_{\zeta-\delta}^{\zeta+\delta}\frac{w}{\varepsilon}\,dx = \int_{-\infty}^{+\infty} w\,d\xi \quad (3.12)$$

This integral represents the quantity of heat emitted per unit of time per unit of the flame surface. For stationary conditions $M = 1$, while in the case of a non-stationary curved flame front, parameter M defines the non-stationary rate of burning. Taking this into consideration the conditions of conservation of flow of heat and the reactant, will be expressed by

$$\varepsilon\frac{\partial}{\partial t}(\rho\theta)^* - [\rho\theta]_{\zeta-0}^{\zeta+0}\frac{\partial\zeta}{\partial t} =$$

$$= \left[-\rho u\theta + \frac{\varepsilon}{\sigma}\frac{\partial\theta}{\partial x}\right]_{\zeta-0}^{\zeta+0} - \varepsilon\frac{\partial}{\partial y}(\rho v\theta)^* - \left[-\rho v\theta + \frac{\varepsilon}{\sigma}\frac{\partial\theta}{\partial y}\right]_{\zeta-0}^{\zeta+0}\frac{\partial\zeta}{\partial y} + M$$

$$\varepsilon\frac{\partial}{\partial t}(\rho s)^* - [\rho s]_{\zeta-0}^{\zeta+0}\frac{\partial\zeta}{\partial t} = \quad (3.13)$$

$$= \left[-\rho u s + \frac{\varepsilon}{\sigma L}\frac{\partial s}{\partial x}\right]_{\zeta-0}^{\zeta+0} - \varepsilon\frac{\partial}{\partial y}(\rho v s)^* - \left[-\rho v s + \frac{\varepsilon}{\sigma L}\frac{\partial s}{\partial y}\right]_{\zeta-0}^{\zeta+0}\frac{\partial\zeta}{\partial y} - M$$

Linearizing the boundary conditions (3.10) to (3.13) for the case of small perturbations we obtain

$$\left[p' + \rho' U^2 + RU\left(2u' - \frac{\partial\zeta}{\partial t}\right)\right]_{-0}^{+0} =$$

$$= \varepsilon\left\{\left[\frac{4}{3}\frac{\partial u'}{\partial x} - \frac{2}{3}\frac{\partial v'}{\partial y}\right]_{-0}^{+0} - \frac{\partial}{\partial t}(Ru')^* - \frac{\partial}{\partial t}(\rho' U)^* - \frac{\partial}{\partial y}(RUv')^*\right\}$$

$$\left[RUv' - P\frac{\partial\zeta}{\partial y}\right]_{-0}^{+0} = \varepsilon\left\{\left[\frac{\partial v'}{\partial x} + \frac{\partial u'}{\partial y}\right]_{-0}^{+0} - \frac{\partial}{\partial t}(Rv')^* - \frac{\partial}{\partial y}(p')^*\right\}$$

$$\left[\rho' U + R\left(u' - \frac{\partial\zeta}{\partial t}\right)\right]_{-0}^{+0} = \varepsilon\left\{-\frac{\partial}{\partial t}(\rho')^* - \frac{\partial}{\partial y}(Rv')^*\right\}$$

$$\left[RU\theta' + \rho' U\Theta + R\Theta\left(u' - \frac{\partial\zeta}{\partial t}\right)\right]_{-0}^{+0} =$$

$$= \varepsilon\left\{\left[\frac{1}{\sigma}\frac{\partial\theta'}{\partial x}\right]_{-0}^{+0} - \frac{\partial}{\partial t}(R\theta')^* - \frac{\partial}{\partial t}(\rho'\Theta)^* - \frac{\partial}{\partial y}(R\Theta v')^*\right\} + M' \quad (3.14)$$

$$\left[RUs' + \rho' US + RS\left(u' - \frac{\partial\zeta}{\partial t}\right)\right]_{-0}^{+0} =$$
$$= \varepsilon\left\{\left[\frac{1}{\sigma L}\frac{\partial s'}{\partial x}\right]_{-0}^{+0} - \frac{\partial}{\partial t}(Rs')^* - \frac{\partial}{\partial t}(\rho' S)^* - \frac{\partial}{\partial y}(RSv')^*\right\} - M'$$

For further computation of the terms denoted by ( )* we shall have to use a definite form of flame structure. We shall assume that the latter is of the form given by the Zeldovich and Frank-Kamenetskii theory [13 and 14]. This theory assumes that the activation energy of the chemical combustion reaction is very great, and that the flame consists of two zones, viz. the preheating zone (its width is of the order of ε), and the comparatively much narrower chemical reaction zone. In accordance with this we shall disregard the width of the reaction zone, and assume that the reaction rate is determined only by the temperature of combustion products leaving this zone ($M = M(\theta_r)$). Generally, the temperature θ_r may differ from the adiabatic temperature of combustion under stationary conditions ($\theta_r \neq \theta = 1$) see for example [15 and 16]. Structure of the flame front is defined by the system of equations (2.1) which comprises the hydrodynamic, diffusion and thermal conductivity equations. In the case of a curved flame front curvilinear orthogonal coordinates originating at the flame front should be used, as the term 'flame structure' means the parameter distribution within the flame front in the direction of the normal n to its surface. We then substitute $\eta = n / \varepsilon$ for n. After this substitution, and because for the computation of the values of terms ( )* it is sufficient to know the flame structure in the zero-order approximation only, all terms of the order of ε can be disregarded. It is easy to see that because of this all derivatives with respect to time and to the coordinate collinear with the flame front, will disappear. All Lamé coefficients will be equal to unity. For flame fronts of small curvature, the direction of its normal will be collinear with the x-axis with the approximation to the higher order terms. We can, therefore, assume that $\eta = n / \varepsilon = \xi = (x - \zeta) / \varepsilon$.

Then, from equations (2.1), we obtain for the area outside the chemical reaction zone

$$\frac{\partial}{\partial\xi}\left(-p - \rho v_n^2 + \frac{4}{3}\frac{\partial v_n}{\partial\xi}\right) = 0, \qquad \frac{\partial}{\partial\xi}\left(-\rho v_n v_\tau + \frac{\partial v_\tau}{\partial\xi}\right) = 0, \qquad \frac{\partial}{\partial\xi}(\rho v_n) = 0$$
$$\frac{\partial}{\partial\xi}\left(-\rho v_n\theta + \frac{1}{\sigma}\frac{\partial\theta}{\partial\xi}\right) = 0, \qquad \frac{\partial}{\partial\xi}\left(-\rho v_n s + \frac{1}{\sigma L}\frac{\partial s}{\partial\xi}\right) = 0 \tag{3.15}$$

where v_n and v_τ are the components of the stream velocity, normal and tangent respectively to the flame front. These are related to velocities u and v by

$$v_n = V_n + v_n' = U + u' - \frac{\partial\zeta}{\partial t}, \qquad v_\tau = v' + U\frac{\partial\zeta}{\partial y} \tag{3.16}$$

The system of equations (3.15) must be supplemented by the equation of state (2.2).

The boundary conditions for the system of equations (3.15) will be: (1) conditions, at infinity (for $\xi \to \pm\infty$ the solutions should coincide with the values of parameters at the flame surface obtained by Landau (formulas (2.6) and (2.8) for $x = 0$)); (2) conditions in the reaction zone. Here the conditions of continuity of the tangential velocity, temperature and concentration must be fulfilled (conditions of conservation in this case will be fulfilled automatically). With the above boundary conditions and use of linearizing procedure, we obtain the solution of system (3.15) in the form

$$U_- = 1 + \frac{1-\alpha}{\alpha}e^{\sigma\xi}, \quad U_+ = \frac{1}{\alpha}, \quad V_- = V_+ = 0 \tag{3.17}$$

$$P_{-} = P_{-\infty} + \frac{1-\alpha}{\alpha}\left(\frac{4}{3}\sigma - 1\right)e^{\sigma\xi}, \quad P_{+} = P_{-\infty} - \frac{1-\alpha}{\alpha}$$

$$\Theta_{-} = e^{\sigma\xi}, \quad \Theta_{+} = 1, \quad S_{-} = 1 - e^{\sigma L\xi}, \quad S_{+} = 0$$

$$R_{-} = 1 \Big/ \left(1 + \frac{1-\alpha}{\alpha} e^{\sigma\xi}\right), \qquad R_{+} = \alpha$$

for stationary conditions, and

$$u_{-}' = \left\{A^0 + \frac{1-\alpha}{\alpha}\left[(A^0 - \omega D)(1 + \sigma\xi) + H^0\right] e^{\sigma\xi}\right\} e^{iy+\omega t}, \quad u_{+}' = (B^0 + C^0)\, e^{iy+\omega t}$$

$$v_{-}' = i\left[A^0 - \left(A^0 + B^0 + \alpha\omega C^0 - \frac{1-\alpha}{\alpha} D\right)e^{\xi} - \frac{1-\alpha}{\alpha} D e^{\sigma\xi}\right] e^{iy+\omega t}$$

$$v_{+}' = -i\,(B^0 + \alpha\omega C^0)\, e^{iy+\omega t}$$

$$p_{-}' = \left\{-A^0(\omega + 1) + \frac{1-\alpha}{\alpha}\left(\frac{4}{3}\sigma - 1\right)\left[(A^0 - \omega D)(2 + \sigma\xi) + H^0\right] e^{\sigma\xi}\right\} e^{iy+\omega t}$$

$$p_{+}' = B^0(\alpha\omega - 1)\, e^{iy+\omega t}, \qquad \rho_{+}' = -\alpha(1-\alpha) H^0 e^{iy+\omega t} \tag{3.18}$$

$$\rho_{-}' = -\frac{1-\alpha}{\alpha}\left[(A^0 - \omega D)\sigma\xi + H^0\right]\left(1 + \frac{1-\alpha}{\alpha} e^{\sigma\xi}\right)^{-2} e^{\sigma\xi}\, e^{iy+\omega t}$$

$$\theta_{-}' = \left\{\left[(A^0 - \omega D)\sigma\xi + H^0\right] e^{\sigma\xi}\right\} e^{iy+\omega t}, \quad \theta_{+}' = H^0 e^{iy+\omega t}$$

$$s_{-}' = -(A^0 - \omega D)\,\sigma L\xi e^{\sigma L\xi} e^{iy+\omega t}, \quad s_{+}' = 0$$

for the perturbations.

Using the formulas (3.17) and (3.18), we can compute the values of the terms ( )* appearing in the boundary conditions (3.14), with the following result

$$(p')^* = \frac{1-\alpha}{\alpha}\frac{1}{\sigma}\left(\frac{4}{3}\sigma - 1\right)(A^0 - \omega D + H^0)\, e^{iy+\omega t}$$

$$(\rho')^* = -\frac{1}{\sigma}\left[(A^0 - \omega D)\ln\alpha + (1-\alpha)H^0\right] e^{iy+\omega t}$$

$$(Ru')^* = \frac{1}{\sigma}\left\{A^0 \ln\alpha - \left[I_1(\alpha) + \ln\alpha\right](A^0 - \omega D) - H^0\ln\alpha\right\} e^{iy+\omega t}$$

$$(Rv')^* = i\left\{\frac{\ln\alpha}{\sigma}A^0 - \frac{\alpha(A^0 + B^0 + \alpha\omega C^0)}{1-\alpha} I_2(\alpha,\sigma) + \left[I_2(\alpha,\sigma) + \frac{\ln\alpha}{\sigma}\right]D\right\} e^{iy+\omega t}$$

$$\left(I_1(\alpha) \equiv \int_0^\infty \frac{te^{-t}\,dt}{\alpha/(1-\alpha) + e^{-t}}, \qquad I_2(\alpha, m) \equiv \int_0^\infty \frac{e^{-t}\,dt}{\alpha/(1-\alpha) + e^{-mt}}\right)$$

$$(R\theta')^* = -\frac{\alpha}{\sigma(1-\alpha)}\left[I_1(\alpha)(A^0 - \omega D) + H^0\ln\alpha\right] e^{iy+\omega t}$$

$$(Rs')^* = \frac{\alpha}{1-\alpha}\frac{1}{\sigma L} I_3\left(\alpha, \frac{1}{L}\right)(A^0 - \omega D)\, e^{iy+\omega t} \left(I_3(\alpha, m) \equiv \int_0^\infty \frac{te^{-t}\,dt}{\alpha/(1-\alpha) + e^{-mt}}\right)$$

$$(\rho' U)^* = \frac{1}{\sigma}\left[I_1(\alpha)(A^0 - \omega D) + H^0\ln\alpha\right] e^{iy+\omega t}$$

$$(\rho'\Theta)^* = \frac{1}{\sigma}\left\{\frac{\alpha}{1-\alpha}\left[I_1(\alpha) + \ln\alpha\right](A^0 - \omega D) + \left(\alpha + \frac{\alpha}{1-\alpha}\ln\alpha\right)H^0\right\} e^{iy+\omega t} \tag{3.19}$$

$$(\rho' S)^* = \frac{1}{\sigma}\left\{\left[-\ln\alpha + \frac{\alpha}{1-\alpha}\frac{1}{L}I_2\left(\alpha, \frac{1}{L}\right) - \frac{\alpha}{1-\alpha}\frac{1}{L}I_3\left(\alpha, \frac{1}{L}\right)\right](A^0 - \omega D) + \right.$$
$$\left. + \left[-1 + \frac{\alpha}{1-\alpha}I_2\left(\alpha, \frac{1}{L}\right)\right]H^0\right\}e^{iy+\omega t}$$

$$(RUv')^* = i\left[-(A^0 + B^0 + \alpha\omega C^0) + \frac{1-\alpha}{\alpha}\left(1 - \frac{1}{\sigma}\right)D\right]e^{iy+\omega t}$$

$$(R\Theta v')^* = i\left\{-\frac{\alpha}{1-\alpha}\frac{\ln\alpha}{\sigma}A^0 - \frac{\alpha}{1-\alpha}\frac{1}{\sigma+1}I_2\left(\alpha, \frac{\sigma}{\sigma+1}\right)(A^0 + B^0 + \alpha\omega C^0) - \right.$$
$$\left. - \left[-\frac{1}{\sigma+1}I_2\left(\alpha, \frac{\sigma}{\sigma+1}\right) + \frac{\alpha}{1-\alpha}\frac{\ln\alpha}{\sigma} + \frac{1}{\sigma}\right]D\right\}e^{iy+\omega t}$$

$$(RSv')^* = i\left\{\frac{\ln\alpha}{\sigma}A^0 - \frac{\alpha}{1-\alpha}\frac{1}{\sigma L}I_2\left(\alpha, \frac{1}{L}\right)A^0 - \frac{\alpha}{1-\alpha}\left[I_2(\alpha, \sigma) - \right.\right.$$
$$\left.\left. - \frac{1}{\sigma L+1}I_2\left(\alpha, \frac{\sigma}{\sigma L+1}\right) + \frac{\ln\alpha}{\sigma} + \frac{1}{\sigma(L+1)}I_2\left(\alpha, \frac{1}{L+1}\right)\right]D\right\}e^{iy+\omega t}$$

which completes the derivation of the boundary conditions at the flame surface in the first-order approximation with respect to ε.

4. We shall now determine the characteristic frequency of the problem. Having derived solutions of equations (2.6), (2.8) and (2.10) which satisfy the boundary conditions (3.14), we shall now equate the coefficients of the zero- and first-order approximations with respect to ε, to obtain two systems of homogeneous, linear algebraic equations. The first system obtained by equating to zero the coefficients of the terms of the zero power is used to determine the constants of the zero-order approximation

$$A^0(\omega - 1) + B^0(\alpha\omega + 1) + 2C^0 - \frac{1-\alpha}{\alpha}H^0 = 0$$
$$-A^0 - B^0 - \alpha\omega C^0 + \frac{1-\alpha}{\alpha}D = 0$$
$$-A^0 + \alpha B^0 + \alpha C^0 + (1-\alpha)\omega D - (1-\alpha)H^0 = 0 \qquad (4.1)$$
$$\alpha B^0 + \alpha C^0 - \alpha\omega D + \alpha H^0 = 0, \quad -A^0 + \omega D = 0$$

It is easily seen that from the last three equations it follows that $H^0 = 0$. Thus, the solution of zero-order approximation does not contain terms related to temperature perturbations downstream of the flame. This is understandable, as the phenomena which can cause temperature perturbations there, i.e. the diffusion and thermal conductivity, are neglected.

Neglecting the fourth equation of (4.1) and assuming $H^0 = 0$, we arrive at the system of equations obtained by Landau (in a somewhat different combination of equations). In order to obtain a nontrivial solution, we equate the determinant of the system to zero, and arrive at Landau's characteristic equation

$$(\Omega^0)^2 + \frac{2\alpha}{1+\alpha}\Omega^0 - \frac{\alpha(1-\alpha)}{1+\alpha} = 0 \qquad (\Omega^0 \equiv \alpha\omega^0) \qquad (4.2)$$

We derive the expressions for the coefficients A^0, B^0 and C^0 in terms of the perturbation amplitude of the surface D, with the assumption that in this system $\omega = \omega^0$. We obtain

$$A^0 = \omega^0 D, \qquad B^0 = \frac{1-\alpha}{\alpha(1+\alpha)}D, \qquad C^0 = \left(\omega^0 - \frac{1-\alpha}{\alpha(1+\alpha)}\right)D \qquad (4.3)$$

Now we shall try to solve the problem in the first-order approximation. Equating coefficient of ε, we obtain

$$(\omega^0-1)A^1+(\alpha\omega^0+1)B^1+2C^1-\frac{1-\alpha}{\alpha}H^1=A^0\omega^0\tau+B^0\alpha\omega^0\tau+\{\ \}_1 \quad (4.4.1)$$

$$-A^1-B^1-\alpha\omega^0C^1=[1-\alpha\omega^0\tau-(\alpha\omega^0)^2]C^0+\{\ \}_2 \quad (4.4.2)$$

$$-A^1+\alpha B^1+\alpha C^1-(1-\alpha)H^1=(1-\alpha)\omega^0\tau D+\{\ \}_3 \quad (4.4.3)$$

$$\alpha B^1+\alpha C^1+\alpha H^1=-\alpha\omega^0\tau D+\{\ \}_4 \quad (4.4.4)$$

$$-A^1=\omega^0\tau D+\{\ \}_5 \quad (4.4.5)$$

A new notation was used in these equations: the right-hand side terms of the boundary condition (3.14) were denoted for brevity by $\varepsilon\{\ \}_k$, where the lower index denotes the equation number. Also, the characteristic frequency of the problem was used in the form given by (1.3), (we also seek the first-order approximation with respect to ε of the characteristic frequency of this problem).

We note that H^1 is easily found from the system (4.4). From (4.4.3), (4.4.4) and (4.4.5) we obtain

$$H^1=-\{\ \}_3+\{\ \}_4+\{\ \}_5 \quad (4.5)$$

We shall use the solution of zero-order approximation i.e. (4.3) for computing the right-hand sides of the system (4.4) in terms of the flame front perturbation amplitude. We obtain for the parameters $(\ )^*$

$$(p')^*=(\rho')^*=(R\theta')^*=(Rs')^*=(\rho'U)^*=(\rho'\Theta)^*=(\rho'S)^*=0$$

$$(Ru')^*=\omega^0\frac{\ln\alpha}{\sigma}De^{iy+\omega t},\quad (Rv')^*=i(\omega^0+1)\frac{\ln\alpha}{\sigma}De^{iy+\omega t}$$

$$(RUv')^*=-i\frac{1-\alpha}{\alpha}\frac{1}{\sigma}De^{iy+\omega t} \quad (4.6)$$

$$(RSv')^*=\frac{i}{\sigma}\left[(\omega^0+1)\ln\alpha-\frac{\alpha}{1-\alpha}\frac{\omega^0}{L}I_2\left(\alpha,\frac{1}{L}\right)+\frac{1}{L+1}I_2\left(\alpha,\frac{1}{L+1}\right)\right]De^{iy+\omega t}$$

$$(R\Theta v')^*=-\frac{i}{\sigma}\left[1+\frac{\alpha(\omega^0+1)}{1-\alpha}\ln\alpha\right]De^{iy+\omega t}$$

and with the use of (4.6) for the right-hand sides of system (4.4) we have

$$\{\ \}_1=-\left[\left(2+\frac{1}{\sigma}\right)\frac{1-\alpha}{\alpha}+\frac{(\omega^0)^2}{\sigma}\ln\alpha\right]D$$

$$\{\ \}_2+[1-(\alpha\omega^0)^2]C^0=-\frac{\omega^0(\omega^0+1)}{\sigma}D\ln\alpha,\qquad \{\ \}_3=\frac{\omega^0+1}{\sigma}D\ln\alpha \quad (4.7)$$

$$\{\ \}_4=-\frac{1}{\sigma}\left[1+\frac{\alpha}{1-\alpha}(\omega^0+1)\ln\alpha\right]D+zH^1$$

$$\{\ \}_5=\frac{1}{\sigma}\left[(\omega^0+1)\ln\alpha-\frac{\alpha}{1-\alpha}\frac{\omega^0}{L}I_2\left(\alpha,\frac{1}{L}\right)+\frac{1}{L+1}I_2\left(\alpha,\frac{1}{L+1}\right)\right]D-zH^1$$

while for H^1 we have

$$H^1 = -\frac{1}{\sigma}\left\{[\ \ ]_a + \frac{\alpha\omega^0}{\alpha - 1}[\ \ ]_b\right\} D = -KD \tag{4.8}$$

where for brevity we use the following notation

$$[\ \ ]_a \equiv \left[1 + \frac{\alpha}{1-\alpha}\ln\alpha - \frac{1}{L-1} I_2\left(\alpha, \frac{1}{L+1}\right)\right], \quad [\ \ ]_b \equiv \left[\frac{1}{L} I_2\left(\alpha, \frac{1}{L}\right) + \ln\alpha\right] \tag{4.9}$$

and utilise the dependence of the reaction rate on temperature, which is

$$M' = z\theta_r' = \varepsilon z H^1 \exp(iy + \omega t)$$

(The terms within the parentheses $[\ ]_{a,b}$ become zero for the Lewis number $L = 1$).

We rearrange our equations so as to present them in a form similar to that of the Landau system [1 and 2]. For this we have to perform the following operations

$$-\frac{2}{\alpha}(4.4.1) + 2\frac{1-\alpha}{\alpha}(4.4.5), \quad -(4.4.2), \quad \frac{1}{\alpha}(4.4.3) - \frac{1}{\alpha}(4.4.5), \quad -(4.4.5)$$

The numbers in parentheses correspond to the equations of the system (4.4). Equation (4.4.4) is omitted, since it was used to obtain H^1. We have

$$(\omega^0 + 1)A^1 + (\alpha\omega^0 - 1)B^1 + \left\{\tau\left[\omega^0 - \frac{1-\alpha}{\alpha(1+\alpha)}\right] + \frac{1-\alpha}{\alpha}\left(2 - \frac{1}{\sigma}\right) + \right.$$
$$\left. + \frac{(\omega^0)^2}{\sigma}\ln\alpha - \frac{1-\alpha}{\alpha}(1-2z)K\right\} D = 0$$
$$A^1 + B^1 + \alpha\omega^0 C^1 - \left\{\alpha\omega^0\tau\left[\omega^0 - \frac{1-\alpha}{\alpha(1+\alpha)}\right] + \frac{\omega^0(\omega^0+1)}{\sigma}\ln\alpha\right\} D = 0 \tag{4.10}$$
$$B^1 + C^1 + \left\{\omega^0\tau + \frac{1}{\alpha\sigma} + \frac{\omega^0 + 1}{(1-\alpha)\sigma}\ln\alpha + \left(1 - \frac{z}{\alpha}\right)K\right\} D = 0$$
$$A^1 + \left\{\omega^0\tau + \frac{1}{\sigma}\left[1 + \frac{\omega^0+1}{1-\alpha}\ln\alpha\right] + (1-z)K\right\} D = 0$$

Here, as in the case of the solution of zero-order approximation, the determinant of system (4.10) must be zero, for a nontrivial solution to exist. This provides the condition for the determination of correction of the characteristic frequency τ of our problem.

We find

$$\tau = \frac{1-\alpha}{1-\alpha-\Omega^0} - \frac{1+\alpha+2\Omega^0}{1-\alpha-\Omega^0}\frac{1}{\sigma}\left(1 + \frac{\ln\alpha}{1-\alpha}\right) +$$
$$+ \frac{1+\Omega^0}{1-\alpha-\Omega^0}(1+\alpha-2z)\frac{1}{\sigma}\left\{[\ \ ]_a + \frac{\Omega^0}{1-\alpha}[\ \ ]_b\right\} \tag{4.11}$$

If (4.11) is extrapolated into the region of wavelengths perturbation for which $\omega = 0$, then the parameter τ becomes equal to the critical Reynolds number, $\tau = N_{R_0}$, taken with respect to the perturbation wavelength, or more precisely with respect to the wave number $k = 2\pi/\lambda$.

5. Formula (4.11) derived in the previous section shows the influence of individual dissipative effects on the flame stability.

The first term of this formula represents the effect of viscosity.

Using for Ω^0 the expression

$$\Omega^0 = (-\alpha + \sqrt{\alpha + \alpha^2 - \alpha^3}) / (1 + \alpha) \tag{5.1}$$

it is easy to prove (see (4.2)) that this term is always positive, therefore, viscosity has always a stabilising effect on the flame. This is in agreement with conclusions arrived at in [5 and 6], but contradicts those of [4 and 8] in which a physically odd, unstabilising effect of viscosity is shown. The critical Reynolds number, as determined by viscosity effects, is

$$N_{R1} = (1 - \alpha) / (1 - \alpha - \Omega^0) \tag{5.2}$$

and is not high. The dependence of N_{R1} on α is shown in Fig. 2. For real flames $\alpha = 0.1$ to 0.2, and $N_{R1} = 1.3$ to 1.5.

The second term of the formula (4.11) represents the effect of thermal conductivity. The critical Peclet number corresponding to this effect only is

$$\Pi_2 = \sigma N_{R2} = -\frac{1 + \alpha + 2\Omega^0}{1 - \alpha - \Omega^0}\left(1 + \frac{\ln \alpha}{1 - \alpha}\right) \tag{5.3}$$

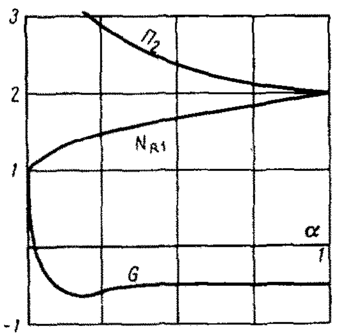

FIG. 2

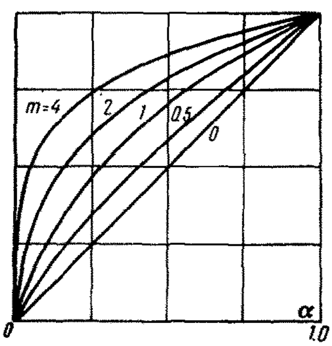

FIG. 3

The dependence of Π_2 on α is also shown on the fig. 2. Thermal conductivity has also only a stabilising effect on the flame. For the real flames the Peclet number is $\Pi_2 \approx 3$.

The last term of the formula (4.11) represent the effect of the relationship between diffusion and thermal conductivity on the flame stability. The critical Peclet number due to this effect is

$$\Pi_3 = \frac{1 + \Omega^0}{1 - \alpha - \Omega^0}(1 + \alpha - 2z)\left\{[\quad]_a + \frac{\Omega^0}{1 - \alpha}[\quad]_b\right\} \tag{5.4}$$

It is dependent on two parameters, viz. the degree of thermal expansion α and the Lewis number L, and in this manner correlates the effects of diffusion and thermal conductivity.

It should be emphasised, however, that here the role of the Lewis number L is somewhat different than in the case of purely diffusive-thermal stability [15 and 16]. In the latter, stability is dependent on the redistribution of heat and matter between the hills and dales of the flame front by means of transverse thermal diffusion and conduction flows, originating at the flame curvatures. In our case the transverse, flow of heat and matter is convective; we analyse the flame front in the vicinity of the boundary layer, diffusion and thermal conductivity are considered in the direction normal to the front only (as quasistationary changes of the concentration and temperature profiles), while the tangential transfer of heat and matter along the front is brought about by the oscillations of the tangential velocity. Therefore the diffusive thermal phenomena, in the sense given in [15 and 16], would only appear in the second-order approximation with respect to ε of the Landau solution. The part which the Lewis number plays in our problem lies in the fact, that the width of the diffusion and thermal fronts differ from each other, hence, the effective values of concentration and temperature, trasported by transverse pulsations, are also different. Fig. 1 shows how the streamlines become distorted, and how the y-components of velocity change their sign within the flame front. Depending on the degree of thermal expansion α, the transport phenomena may predominate either downstream, or upstream of the flame front, and the resulting effect will be different, even though relationship between the diffusion and thermal conductivity remains the same.

This can be easily proved. The Peclet number for $\alpha \to 0$ (considerable thermal expansion) is

$$\Pi_3 \to (1-2z)(L-1)/L \approx -2z(L-1)/L \tag{5.5}$$

while for $\alpha \to 1$ (no thermal expansion), we have

$$\Pi_3 \to -2(1-z)(L-1)/L(L+1) \approx 2z(L-1)/L(L+1) \tag{5.6}$$

from which we see that sign of $L-1$ remains the same, when the sign of the Peclet number changes.

If the Lewis number is close to unity, then $|L-1| \ll 1$, and (5.4) can be replaced with the simpler equation

$$\Pi_3 \approx -2z\frac{1+\Omega^0}{1-\alpha-\Omega^0}\left[1-\frac{\Omega^0+\alpha}{1-\alpha}I_1(\alpha)\right](L-1) = -2zG(\alpha)(L-1) \tag{5.7}$$

Function $G(\alpha)$ is shown on fig. 2. It becomes negative for very small α, but this does not occur in practice.

In the general case formula (5.4) should be used. In order to facilitate computations with the aid of this formula curves of $[\alpha/(1-\alpha)]\,I_2(\alpha, m)$ for several values of the parameter m are given on fig. 3.

Thus, the effect of transport phenomena is that of stabilising small perturbations. The critical Reynolds number τ obtained by the extrapolation of the linear correction does not greatly differ from unity. For example, for $\alpha = 0.2$, $z = 10$, $\sigma = 1$, and $L = 1.2$, we have $\tau \approx 7$.

In concluding we wish to express out thanks to G.I. Bareblatt and Ia.B. Zel'dovich for the formulation of the problem and its discussion and to O.I. Leipunovski and V.I. Iagodkin for their valuable remarks

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